2024 IBS-DIMAG workshop on combinatorics and geometric measure theory

Sunday, July 14, 2024

14:00-17:00		Meeting and discussion

Monday, July 15, 2024

9:00—9:30		Registration, coffee
9:30—9:40	Doowon Koh	Opening remarks
9:40—10:40	János Pach	With strings attached
10:40—11:00		Coffee break
11:00—11:30	Bochen Liu	Dimension of Diophantine approximation, orthogonal projection, and sum-product
11:30—12:00	Giorgis Petridis	Products of differences in arbitrary finite fields
12:00—14:00		Lunch
14:00—14:30	Manik Dhar	Furstenberg sets for higher dimensional flats over F_q and R
14:30—15:00	Jonathan Pakianathan	Random walks and the "Euclidean" association scheme in finite vector spaces
15:00—15:30		Coffee break
15:30—17:00		Open Problem Session

Tuesday, July 16, 2024

9:30—10:30	Izabella Łaba	A short survey of integer tilings
10:30—11:00		Coffee break
11:00—11:30	Mihalis Kolountzakis	Common fundamental domains
11:30—12:00	Alexander Clifton	Covers of triangular grids
12:00—14:00		Lunch
14:00—14:30	Eyvindur Palsson	Geometric averaging operators and point configurations
14:30—15:00	Steven Senger	Falconer-type problems for dot products
15:00—15:30		Coffee break
15:30—17:30		Small group collaboration

Wednesday, July 17, 2024

9:30—10:30	Pertti Mattila	Hausdorff dimension of plane sections and general intersections
10:30—11:00		Coffee break
11:00—11:30	Wei-Hsuan Yu	On the size of maximal binary codes with 2, 3, and 4 distances
11:30—12:00	Hai Long Dao	The combinatorics of syzygies
12:00—14:00		Lunch
14:00—19:00		Excursion
19:00		Banquet

Thursday, July 18, 2024

9:30—10:30	Hong Wang	Some structure of Kakeya sets in R^3
10:30—11:00		Coffee break
11:00—11:30	Alan Chang	Dividing a set in half
11:30—12:00	Terry Harris	Subsets of vertical planes in the first Heisenberg group
12:00—14:00		Lunch
14:00—14:30	Charlotte Aten	A multi-linear geometric estimate
14:30—15:00	Semin Yoo	Improved upper bounds for the largest size of Diophantine m-tuples
15:00—15:30		Coffee break
15:30—17:30		Small group collaboration

Friday, July 19, 2024

9:30—10:30	Cosmin Pohoata	The Heilbronn triangle problem
10:30—11:00		Coffee break
11:00—11:30	Andreas Holmsen	Colorful intersections and Tverberg partitions
11:30—12:00	Jinha Kim	Star clusters in independence complexes of hypergraphs
12:00—14:00		Lunch
14:45—15:15	Olivine Silier	A Proto Inverse Szemerédi–Trotter Theorem
15:15—15:45	Matthew Kroeker	The Average Number of Points in a Spanned Plane
15:45—16:15		Coffee break
16:15—17:30		Small group collaboration

Monday, July 15

9:40—10:40 Plenary Lecture
Chair: Doowon Koh

Speaker: János Pach
Title: With strings attached
Abstract: The intersection graph of a collection C of sets is the graph whose vertex set is C and in which two sets in C are connected by an edge if and only if they have nonempty intersection. String graphs, intersection graphs of continuous curves ("strings") in the plane have been studied intensively since the 1960s, for their exciting algorithmic and combinatorial properties and their applications in chip design, network theory, graph drawing and elsewhere. After giving a whirlwind tour of string graph theory, I will present some recent results and annoying open problems. In particular, I will sketch the proof of the following theorem, joint with Jacob Fox and Andrew Suk. Given a set R of n red curves, and and a set B of n blue curves in the plane such that any two of them meet at most once, there are subsets $R' \subset R$ and $B' \subset B$ with $|R'|, |B'| \geq \Omega(n)$ with the property that either every curve in R' crosses every curve in B', or every curve in R' is disjoint from every curve in B'.

11:00—12:00 Contributed Talk Session
Chair: Ben Lund

Speaker: Bochen Liu
Title: Dimension of Diophantine approximation, orthogonal projection, and sum-product
Abstract: Given $0 < s < 1$, we construct a new family of sets in the line of Hausdorff dimension s via Diophantine approximation, in which the classical Jarnik example is the endpoint case. Then we are able to show a recent result of Ren and Wang, known sharp on orthogonal projection, is also sharp on $A + cB, c \in C$. This is joint work with Longhui Li.

Speaker: Giorgis Petridis
Title: Products of differences in arbitrary finite fields.
Abstract: It is not too hard to prove that any subset A of a finite field with q elements that has size $q^{2/3}$ the set $(A - A)(A - A)$ contains at least $q/2$ elements. We describe the ingredients needed to relax the condition to $q^{2/3-c}$ for a small absolute $c > 0$. Old joint work with Brendan Murphy.

14:00—15:00 Contributed Talk Session
Chair: Semin Yoo

Speaker: Manik Dhar
Title: Furstenberg sets for higher dimensional flats over F_q and R
Abstract: A Kakeya set in R^n is a set that contains a line segment in every direction. The Kakeya conjecture states that these sets have dimension n (open for $n \geq 3$). Over F_q^n, a Kakeya set is similarly defined as containing a line in every direction. A breakthrough by

Zeev Dvir using the polynomial method showed that Kakeya sets must have size $C_n q^n$. In this talk, we consider some works on a generalization of this problem for higher dimensional flats. Furstenberg sets in F_q^n and R^n are sets that have large intersections with k flats in every direction. For $k \geq 2$ in F_q^n and $k > log_2 n$ for R^n, we show that these sets are large and give a very simple description of all tight examples. These results over finite fields have recently had surprising applications in the study of lattice coverings and linear hash functions. Based on works with Zeev Dvir, Ben Lund, and upcoming work with Paige Bright.

Speaker: Jonathan Pakianathan
Title: Random walks and the "Euclidean" association scheme in finite vector spaces
Abstract: We will discuss the theory of finite association schemes and talk about a particular association scheme arising from "distance-t" graphs over finite vector spaces V of odd order equipped with the "Euclidean" quadratic form. This "Euclidean" association scheme was studied in prior work of W.M.Kwok, E. Bannai, O. Shimabukuro and H. Tanaka as well as in Brittenham's Ph.D. Thesis.
We will explain how vertical equidistribution of Kloosterman sums, is equivalent to certain asymptotics in the probability of return to origin in a k-step distance-t random walk in these vector spaces as well as outline how Delsarte's linear programming bound can constrict distance sets of subsets E of the vector space.
This is based on joint work with Charles Brittenham.

Tuesday, July 16

9:30—10:30 Invited lecture
Chair: Chun-Yen Shen

Speaker: Izabella Łaba
Title: A short survey of integer tilings
Abstract: A set $A \subset \mathbb{Z}$ tiles the integers by translations if there is a set $T \subset \mathbb{Z}$ such that every integer $n \in \mathbb{Z}$ has a unique representation $n = a + t$ with $a \in A$ and $t \in T$. The main open question regarding integer tilings is the Coven-Meyerowitz conjecture, providing a tentative characterization of finite tiles. We will survey some of the recent developments and open questions in this area, including a very recent joint result with Itay Londner where we prove the Coven-Meyerowitz tiling conditions for a new class of tilings.

11:00—12:00 Contributed talk session
Chair: Eyvindur Palsson

Speaker: Mihalis Kolountzakis
Title: Common fundamental domains
Abstract: A fundamental domain T of a group H in a larger, abelian, group G is a selection of one representative from each coset of H in G. In other words $G = H \oplus T$ with the sum being direct. The main theme of this talk is when a collection $H_1, \ldots, H_n$ of

subgroups of G, all of the same index, admit a common fundamental domain T

$$H_1 \oplus T = H_2 \oplus T = \cdots = H_n \oplus T = G.$$

In tiling language we are seeking a set $T \subseteq G$ that will tile G when translated by any of the subgroups $H_1, \ldots, H_n$
This problem has many different aspects: analytic, geometric, combinatorial and purely algebraic. We hope to be able to show several of these, explain some of the recent progress.

Speaker: Alexander Clifton
Title: Covers of triangular grids
Abstract: We consider the minimum number, $f(n, d, k)$, of affine hyperplanes in $\mathbb{R}^d$ needed to cover all the points of the triangular grid
$T_d(n) := \{(x_1, \ldots, x_d) \in \mathbb{Z}^d_{\geq 0} \mid x_1 + \cdots + x_d \leq n - 1\}$ at least k times each. Combining linear programming with combinatorial techniques, we determine the exact answer for $d = 2$ and $k \leq 4$, as well as an asymptotic formula in terms of n whenever $d \geq k - 2$. Joint work with Abdul Basit and Paul Horn.

14:00—15:00 Contributed Talk Session
Chair: Doowon Koh

Speaker: Eyvindur Palsson
Title: Geometric averaging operators and point configurations
Abstract: Two classic questions focus on distance: the Erdős distinct distance problem, which asks about the least number of distinct distances determined by points in the plane, and its continuous analog, the Falconer distance problem. When studying the Falconer distance problem, a geometric averaging operator, namely the spherical averaging operator, arises naturally. Questions similar to the Erdős distinct distance problem and the Falconer distance problem can also be posed for more complicated point configurations and correspondingly new geometric averaging operators appear. In this talk I will give a brief introduction to the motivating point configuration questions and then report on some novel geometric averaging operators and their mapping properties.

Speaker: Steven Senger
Title: Falconer-type problems for dot products
Abstract: The celebrated Falconer distance problem asks for a threshold on the Hausdorff dimension of a subset $E \subseteq \mathbb{R}^d$ that will guarantee the set of distances determined by pairs of points in E has positive Lebesgue measure. We discuss a variant of this problem involving dot products in place of distances.

Wednesday, July 17

9:30–10:30 Plenary lecture
Chair: Yujia Zhai

Speaker: Pertti Mattila
Title: Hausdorff dimension of plane sections and general intersections
Abstract: I shall discuss conditions on a general family $P_\lambda : \mathbb{R}^n \to \mathbb{R}^m, \lambda \in \Lambda$, of orthogonal projections and a measure ω on Λ which guarantee that the Hausdorff dimension formula $\dim A \cap P_\lambda^{-1}\{u\} = s - m$ holds for ω almost all λ for measurable sets $A \subset \mathbb{R}^n$ with positive and finite s-dimensional Hausdorff measure, $s > m$. I shall present some families of projections where this applies. This leads to some new results on the Hausdorff dimension of intersections $\dim A \cap (g(B) + z)$ for almost all rotations g and for positively many $z \in \mathbb{R}^n$.

11:00–12:00 Contributed talk session
Chair: Chun-Yen Shen

Speaker: Hai-Long Dao
Title: The combinatorics of syzygies
Abstract: A fundamental problem in commutative algebra and algebraic geometry is to understand complexity of syzygies, the set of relations between a collection of polynomials. When the polynomials are just monomials, the problems can be reinterpreted combinatorially. I will discuss some fascinating features in this line of research, highlighting unexpected connections to fractals, geodesics and Hirsch-type bounds on diameter of graphs arising from algebraic constructions.

Speaker: Wei-Hsuan Yu
Title: On the size of maximal binary codes with 2, 3, and 4 distances
Abstract: We address the maximum size of binary codes and binary constant weight codes with few distances. Previous works established a number of bounds for these quantities as well as the exact values for a range of small code lengths. As our main results, we determine the exact size of maximal binary codes with two distances for all lengths $n \geq 6$ as well as the exact size of maximal binary constant weight codes with 2,3, and 4 distances for several values of the weight and for all but small lengths.

Thursday, July 18

9:30–10:30 Invited lecture
Chair: Changkeun Oh

Speaker: Hong Wang
Title: Some structure of Kakeya sets in R^3
Abstract: A Kakeya set in R^n is a set of points that contains a unit line segment in every direction. We study the structure of Kakeya sets in R^3 and show that for any Kakeya set K, there exists well-separated scales $0 < \delta < \rho \leq 1$ so that the δ-neighborhood of K is almost as large as the ρ-neighborhood of K. As a consequence, every Kakeya set in R^3 has Assouad dimension 3. This is joint work with Josh Zahl.

11:00–12:00 Contributed talk session
Chair: Izabella Laba

Speaker: Alan Chang
Title: Dividing a set in half
Abstract: We discuss the following isoperimetric-type problem: Given a set E in $\mathbb{R}^d$ with finite volume, is it possible to find an hyperplane P that cuts E in two parts with equal volume, and such that the area of the slice $\mathcal{H}^{d-1}(P \cap E)$ is of the expected order, namely $|E|^{(d-1)/d}$? It turns out that this question has some connections with Radon transforms, Sobolev spaces, and Kakeya sets. This is work in progress with Giovanni Alberti and Gian Maria Dall'Ara.

Speaker: Terry Harris
Title: Subsets of vertical planes in the first Heisenberg group
Abstract: I will present the following result: If A is a subset of the first Heisenberg group which is contained in a vertical plane, then the Hausdorff dimension of A almost surely does not decrease under vertical projection. This is a (very) special case of a Conjecture of Balogh, Durand-Cartagena, Fassler, Mattila, and Tyson, from 2012. The proof uses Fourier analysis. I will discuss some of the methods used, and why the generalisation to arbitrary sets (not contained in a vertical plane) is difficult.

14:00–15:00 Contributed Talk Session
Chair: Jonathan Pakianathan

Speaker: Charlotte Aten
Title: A multi-linear geometric estimate
Abstract: I will discuss some joint work with Alex Iosevich in which we generalized a 2008 result of Hart and Iosevich on level sets of nondegenerate bilinear forms over finite fields. We found that if $E \subset \mathbb{F}_q^d$ is sufficiently large and ϖ is a non-degenerate multi-linear form then ϖ will attain all possible nonzero values as its arguments vary over E, under a certain quantitative assumption on the extent to which E is projective. We then discovered that our bound, which depends on the arity n of the form as well as d, is only nontrivial for $n = 2$ or when $n = 3$ and $d = 3$. Moreover, the cases where our bound may be applied are also subject to the Cauchy–Davenport theorem. I will outline how we may proceed to produce more interesting bounds using the same techniques as in this initial result.

Speaker: Semin Yoo
Title: Improved upper bounds for the largest size of Diophantine m-tuples
Abstract: A set $\{a_1, a_2, \ldots, a_m\}$ of distinct positive integers is a *Diophantine m-tuple* with property $D_k(n)$ if the product of any two distinct elements in the set is n less than a k-th power. One may wonder what is the largest size, $M_k(n)$, of such a tuple. In this talk, we provide a substantial improvement on a recent result by Dixit, Kim, and Murty (2022) on the upper bound of $M_k(n)$. In particular, we show $M_k(n) = o(\log n)$ for a specially chosen sequence of k and n tending to infinity, breaking the $\log n$ barrier unconditionally. Our proof is a combination of Stepanov's method and Gallagher's larger sieve.

This is joint work with Seoyoung Kim, and Chi Hoi Yip.

Friday July 19

9:30—10:30 Invited Lecture **Chair:** Ben Lund

Speaker: Cosmin Pohoata
Title: The Heilbronn triangle problem
Abstract: The Heilbronn triangle problem is a classical problem in discrete geometry with several new connections to various topics in extremal and additive combinatorics, incidence geometry, harmonic analysis, and projection theory. In this talk, we will give an overview of some of these connections, and discuss some recent developments. Based on joint work with Alex Cohen and Dmitrii Zakharov.

11:00—12:00 Contributed talk session
Chair: Minki Kim

Spearker: Andreas Holmsen
Title: Colorful intersections and Tverberg partitions
Abstract: We prove a theorem that interpolates between the colorful Helly theorem and Tverberg's theorem, which are fundamental results in discrete geometry. As a corollary, we also obtain a result concerning r-dimensional transversals to families of convex sets in d-dimensional Euclidean space, which extends and simplifies earlier work of Karasev and Montejano. The proofs involve techniques from discrete geometry, equivariant topology, and topological combinatorics. This is joint work with Michael G. Dobbins and Dohyeon Lee.

Speaker: Jinha Kim
Title: Star clusters in independence complexes of hypergraphs
Abstract: In 2013, Barmak introduced the concept of star clusters in independence complexes of graphs. We generalize Barmak's result, providing an analogous result for the independence complexes of hypergraphs. This implies that for a hypergraph H with a vertex v that is not isolated and not contained in a Berge cycle of length 3, there exists a hypergraph H' with fewer vertices than H such that the independence complex of H is homotopy equivalent to the suspension of the independence complex of H'. As an application, we also prove that if a hypergraph H has no Berge cycle of length divisible by 3, then the sum of all reduced Betti numbers of its independence complex is at most 1.

14:00—15:00 Contributed talk session
Chair: Rutger Campbell

Speaker: Olivine Silier
Title: A Proto Inverse Szemerédi–Trotter Theorem

Abstract: A point-line **incidence** is a point-line pair such that the point is on the line. The **Szemerédi-Trotter** theorem says the number of point-line incidences for n (distinct) points and lines in $\mathbb{R}^2$ is tightly upperbounded by $O(n^{4/3})$. We advance the **inverse problem**: we geometrically characterize 'sharp' examples which saturate the bound by proving the existence of a nice cell decomposition we call the *two bush cell decomposition*. The proof crucially relies on the *crossing number inequality* from graph theory and has a traditional analysis flavor.
Our two bush cell decomposition also holds in the analogous point-unit circle incidence problem. This constitutes an important step towards obtaining an ϵ improvement in the **unit-distance** problem.

Speaker: Matthew Kroeker
Title: The Average Number of Points in a Spanned Plane
Abstract: In 1941, Melchior proved that, for any finite arrangement of points in the plane which is not contained in a line, the average number of points in a spanned line is less than three. In this talk, we discuss the problem of extending Melchior's Theorem to higher dimensions. We show that, for any finite arrangement of points in space which is not contained in a plane nor in the union of two lines, the average number of points in a spanned plane is at most an absolute constant. We also present a generalization of this result to hyperplanes of any dimension.
This talk is based on joint work with Rutger Campbell, Jim Geelen and Ben Lund.